Sets *

Basic Definitions, Operations on sets

Basic Definitions: We think of a set as a collection of objects. The objects that belong to a set are called the elements or members of the set. In algebra, the elements of a set are usually numbers. Sets are commonly written using set braces, { }. For example, the set containing the elements 1, 2, 3, and 4 is written

$$\{1, 2, 3, 4\}.$$

Since the order in which the elements are listed is not important, this same set can also be written as $\{4, 3, 2, 1\}$ or with any other arrangement of the four numbers. To show that 4 is an element of the set $\{1, 2, 3, 4\}$, we use the symbol $\in$ and write

$$4 \in \{1, 2, 3, 4\}.$$

Also, $2 \in \{1, 2, 3, 4\}$. To show that 5 is *not* an element of the set, we place a slash through the symbol:

$$5 \notin \{1, 2, 3, 4\}.$$

Now try Exercises 17 and 19.

It is customary to name sets with capital letters. If S is used to name the set above, then

$$S = \{1, 2, 3, 4\}.$$

Set S was written by listing its elements. It is sometimes easier to describe a set in words. For example, set S might be described as "the set containing the first four counting numbers." In this example, the notation $\{1, 2, 3, 4\}$, with the elements listed between set braces, is briefer than the verbal description. However, the set F, consisting of all fractions between 0 and 1, could not be described by listing its elements. (Try it.)

Set F is an example of an **infinite set,** one that has an unending list of distinct elements. A **finite set** is one that has a limited number of elements. Some infinite sets, unlike F, can be described by listing. For example, the set of numbers used for counting, called the **natural numbers** or the **counting numbers,** can be written as

$$N = \{1, 2, 3, 4, \ldots\},$$

*Preuzeto iz knjige "College Algebra", autori Lial, Hornsby, Schneider where the three dots that the list of elements of the set continues according to the established pattern.

Now try Exercises 9 and 11.

Sets are often written using a variable. For example,

$$\{x|x \text{ is a natural number between 2 and 7}\}$$

(read "the set of all elements x such that x is a natural number between 2 and 7") represents the set $\{3, 4, 5, 6\}$. The numbers 2 and 7 are *not* between 2 and 7. The notation used here, $\{x|x$ is a natural number between 2 and 7$\}$, is called **setbuilder notation.**

EXAMPLE 1: Listing the Elements of a Set

Write the elements belonging to each set.

(a) $x|x$ is a counting number less than 5

(b) $x|x$ is a state that borders Florida

Solution

(a) The counting numbers less than 5 make up the set $\{1, 2, 3, 4\}$.

(b) The states bordering Florida make up the set {Alabama, Georgia}.

Now try exercise 7.

When discussing a particular situation or problem, we can usually identify a **universal set** (whether expressed or implied) that contains all the elements appearing in any set used in the given problem. The letter U is used to represent the universal set. At the other extreme from the universal set is the **null set,** or **empty set,** the set containing no elements. The set of all people twelve feet tall is an example of the null set. We write the null set in either of two ways: using the special symbol $\emptyset$ or else writing set braces enclosing no elements, { }.

CAUTION Do not combine these symbols; $\{\emptyset\}$ is *not* the null set.

Every element of the set $S = \{1, 2, 3, 4\}$ is a natural number. Because of this, set S is a *subset* of the set N of natural numbers, written $S \subseteq N$. By definition, set A is a **subset** of set B if every element of set A is also an element of set B. For example, if $A = \{2, 5, 9\}$ and $B = \{2, 3, 5, 6, 9, 10\}$, then $A \subseteq B$. However, there are some elements of B that are not in A, so B is not a subset of A, written $B \nsupseteq A$. By the definition, every set is a subset of itself. Also, by definition, $\emptyset$ is a subset of every set.

If A is any set, then $\emptyset \subseteq A$.

Figure 1 shows a set A that is a subset of set B. The rectangle in the drawing represents the universal set U. Such diagrams are called **Venn diagrams.**

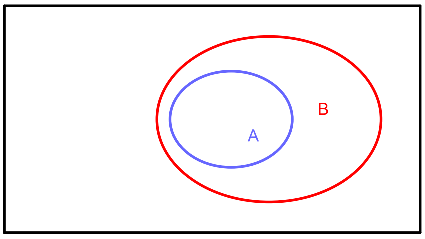


Figure 1

Now try Exercises 55, 59, 63.

Two sets A and B are equal whenever $A \subseteq B$ and $B \subseteq A$. In other words, $A = B$ if the two sets contain exactly the same elements. For example,

$$\{1, 2, 3\} = \{3, 1, 2\}$$

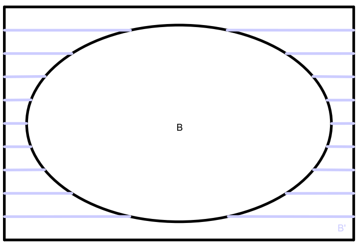
since both sets contain exactly the same elements. However,

$$\{1, 2, 3\} \neq \{0, 1, 2, 3\},$$

since the set $\{0, 1, 2, 3\}$ contains the element 0, which is not an element of $\{1, 2, 3\}$.

Now try Exercises 35 and 37.

Operations on sets: Given a set A and a universal set U, the set of all elements of U that do not belong to set A is called the **complement** of set A. For example, if set A is the set of all the female students in your class, and U is the set of all students in the class, then the complement of A would be the set of all the male students in the class. The complement of set A is written A' (read "A-prime"). The Venn diagram in Figure 2 shows a set B. Its complement, B', is in color.



EXAMPLE 2: Finding the Complement of a Set
Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5, 7\}$, and $B = \{3, 4, 6\}$. Find each set.

(a) A'

(b) B'

(c) $\emptyset'$

(d) U'

Solution

(a) Set A' contains the elements of U that are not in A: $A' = \{2, 4, 6\}$.

(b) $B' = \{1, 2, 5, 7\}$

(c) $\emptyset' = U$

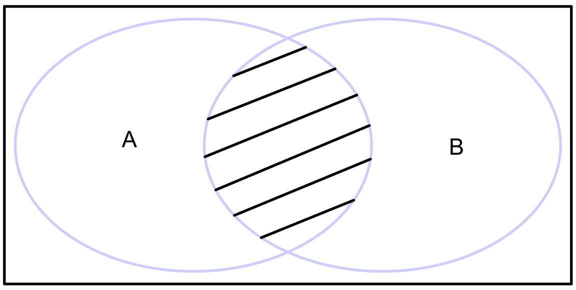
(d) $U' = \emptyset$

Now try Exercise 79.

Given two sets A and B, the set of all elements belonging both to set A *and* to set B is called the **intersection** of the two sets, written $A \cap B$. For example, the elements that belong to both $A = \{1, 2, 4, 5, 7\}$ and $B = \{2, 4, 5, 7, 9, 11\}$ are 2, 4, 5, and 7, so

$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}.$$

The Venn diagram in Figure 3 shows two sets A and B; their intersection, $A \cap B$, is in color.



A∩B

EXAMPLE 3: Finding the Intersection of Two Sets
Find each of the following.

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\}$

(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$

Solution

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\} = \{15, 25\}$ The elements 15 and 25 are the only ones belonging to both sets.

(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\} = \{2, 3, 4\}$

Now try Exercises 41 and 71.

Two sets that have no elements in common are called **disjoint sets.** For example, there are no elements common to both $\{50, 51, 54\}$ and $\{52, 53, 55, 56\}$, so these two sets are disjoint, and

$$\{50, 51, 54\} \cap \{52, 53, 55, 56\} = \emptyset.$$

This result can be generalized.

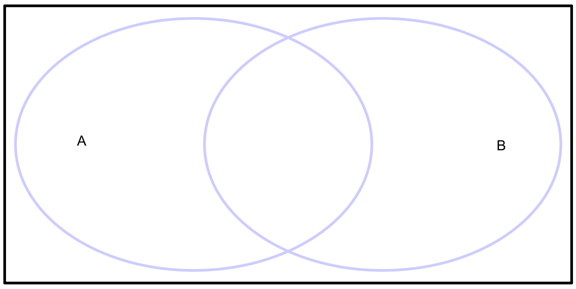
Disjoint sets If A and B are any two disjoint sets, then $A \cap B = \emptyset$

Now try Exercise 75.

The set of all elements belonging to set A *or* to set B (or to both) is called the **union** of the two sets, written $A \cup B$. For example,

$$\{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}.$$

The Venn diagram in Figure 4 shoes two sets A and B; their union $A \cup B$ is in color.



A∪B

EXAMPLE 4: Finding the Union of Two Sets

(a) $\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\}$

(b) $\{1, 3, 5, 7\}' \cup \{2, 4, 6\}$

Solution

(a) Begin by listing the elements of the first set, $\{1, 2, 5, 9, 14\}$. Then include any elements from the second set that are not already listed. Doing this gives

$$\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\} = \{1, 2, 3, 4, 5, 8, 9, 14\}.$$

(b) $\{1, 3, 5, 7\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 5, 6, 7\}$

Now try Exercises 43 and 73.

The processes of finding the complement of a set, the intersection of two sets, and the union of two sets are called **set operations.** These operations, summarized below, are similar to operations on numbers, such as addition, subtraction, multiplication, or division.

Set operations:

For all sets A and B, with universal set U: The **complement** of set A is the set A' of all elements in the universal set that do not belong to set A.

$$\mathbf{A}' = \{\mathbf{x} | \, \mathbf{x} \in \mathbf{U}, \mathbf{x} \notin \mathbf{A}\}$$

The **intersection** of sets A and B, written $A \cap B$, is made up of all the elements belonging to both set A and set B.

$$\mathbf{A} \cap \mathbf{B} = \{\mathbf{x} | \, \mathbf{x} \in \mathbf{A} \wedge \mathbf{x} \in \mathbf{B}\}$$

The **union** of sets A and B, written $A \cup B$, is made up of all the elements belonging to set A or to set B.

$$\mathbf{A} \cup \mathbf{B} = \{\mathbf{x} | \, \mathbf{x} \in \mathbf{A} \vee \in \mathbf{B}\}$$

Appendix Exercises

Use set notation, and list all the elements of each set. See Example 1.

1. $\{12, 13, 14, \ldots, 20\}$

2. $\{8, 9, 10, \ldots, 17\}$

3. $\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{32}\}$

4. $\{3, 9, 27, \ldots, 729\}$

5. $\{17, 22, 27, \ldots, 47\}$

6. $\{74, 68, 62, \ldots, 38\}$

7. {all natural numbers greater than 7 and less than 15}

8. {all natural numbers not greater than 4}

Identify the sets in Exercises 9-16 as finite *or* infinite.

9. $\{4, 5, 6, \ldots, 15\}$

10. $\{4, 5, 6, \ldots\}$

11. $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\}$

12. $\{0, 1, 2, 3, 4, 5, \ldots, 75\}$

13. $\{x | x$ is a natural number larger than 5$\}$

14. $\{x | x$ is a person alive now$\}$

15. $\{x | x$ is a fraction between 0 and 1$\}$

16. $\{x | x$ is an even natural number$\}$

Complete the blanks with either $\in$ or $\notin$ so that the resulting statement is true.

17. 6 ---------- {3,4,5,6}

18. 9 __________ {3,2,5,9,8}

19. -4 __________ {4,6,8,10}

20. -12 __________ {3,5,12,14}

21. 0 __________ {2,0,3,4}

22. 0 __________ {5,6,7,8,10}

23. {3} __________ {2,3,4,5}

24. {5} __________ {3,4,5,6,7}

25. {0} __________ {0,1,2,5}

26. {2} __________ {2,4,6,8}

27. 0 __________ $\emptyset$

28. $\emptyset$ __________ $\emptyset$

Tell whether each statement is true *or* false.

29. $3 \in \{2, 5, 6, 8\}$

30. $6 \in \{-2, 5, 8, 9\}$

31. $1 \in \{3, 4, 5, 11, 1\}$

32. $12 \in \{18, 17, 15, 13, 12\}$

33. $9 \notin \{2, 1, 5, 8\}$

34. $3 \notin \{7, 6, 5, 4\}$

35. {2,5,8,9}={2,5,9,8}

36. {3,0,9,6,2}={2,9,0,3,6}

37. {5,8,9}={5,8,9,0}

38. {3,7,12,14}={3,7,12,14,0}

39. $\{x|x$ is a natural number less than 3}={1,2}

40. $\{x|x$ is a natural number greater than 10}={11,12,13,...}

41. $\{5, 7, 9, 19\} \cap \{7, 9, 11, 15\} = \{7, 9\}$

42. $\{8, 11, 15\} \cap \{8, 11, 19, 20\} = \{8, 11\}$

43. $\{2, 1, 7\} \cup \{1, 5, 9\} = \{1\}$

44. $\{6, 12, 14, 16\} \cup \{6, 14, 19\} = \{6, 14\}$

45. $\{3, 2, 5, 9\} \cap \{2, 7, 8, 10\} = \{2\}$

46. $\{8, 9, 6\} \cup \{9, 8, 6\} = \{8, 9\}$

47. $\{3, 5, 9, 10\} \cap \emptyset = \{3, 5, 9, 10\}$

48. $\{3, 5, , 10\} \cup \emptyset = \{3, 5, 9, 10\}$

49. $\{1, 2, 4\} \cup \{1, 2, 4\} = \{1, 2, 4\}$

50. $\{1, 2, 4\} \cap \{1, 2, 4\} = \emptyset$

51. $\emptyset \cup \emptyset = \emptyset$

52. $\emptyset \cap \emptyset = \emptyset$

Tell whether each statement is true or false.

53. $A \subseteq U$

54. $C \subseteq U$

55. $D \subseteq B$

56. $D \subseteq A$

57. $A \subseteq B$

58. $B \subseteq C$

59. $\emptyset \subseteq A$

60. $\emptyset \subseteq \emptyset$

61. $\{4, 8, 10\} \subseteq B$

62. $\{0, 2\} \subseteq D$

63. $B \subseteq D$

64. $A \nsubseteq C$

Insert $\subseteq$ or $\nsubseteq$ in each blank to make the resulting statement true.

65. {2,4,6} __________ {3,2,5,4,6}

66. {1,5} __________ {0,-1,2,3,1,5}

67. {0,1,2} __________ {1,2,3,4,5}

68. {5,6,7,8} __________ {1,2,3,4,5,6,7}

69. $\emptyset$__________$\{1, 4, 6, 8\}$

70. $\emptyset$__________$\emptyset$

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}, M = \{0, 2, 4, 6, 8\}, N = \{1, 3, 5, 7, 9, 11, 13\}, Q = \{0, 2, 4, 6, 8, 10, 12\}$, *and* $R = \{0, 1, 2, 3, 4\}$. *Use these sets to find each of the following. Identify any disjoint sets. See Examples 2-4.*

71. $M \cap R$

72. $M \cup R$

73. $M \cup N$

74. $M \cap U$

75. $M \cap N$

76. $M \cup Q$

77. $N \cup R$

78. $U \cap N$

79. N'

80. Q'

81. $M' \cap Q$

82. $Q \cap R'$

83. $\emptyset \cap R$

84. $\emptyset \cap Q$

85. $N \cup \emptyset$

86. $R \cup \emptyset$

87. $(M \cap N) \cup R$

88. $(N \cup R) \cap M$

89. $(Q \cap M) \cup R$

90. $(R \cup N) \cap M'$

91. $(M' \cup Q) \cap R$

92. $Q \cap (M \cup N)$

93. $Q' \cap (N' \cap U)$

94. $(U \cap \emptyset') \cup R$

Let U={all students in this school}, M={all students taking this course}, N={all students taking calculus}, *and* P={all students taking history}.
Describe set in words.

95. M'

96. $M \cup N$

97. $N \cap P$

98. $N' \cap P'$

99. $M \cup P$

100. $P' \cup M'$